internal force P_b and undergo the same deformation δ_b. Therefore,

$$\delta_b = +\frac{P_b L_b}{A_b E_b} = +\frac{P_b(18 \text{ in.})}{\frac{1}{4}\pi(0.75 \text{ in.})^2(29 \times 10^6 \text{ psi})} = +1.405 \times 10^{-6} P_b \quad \textbf{(1)}$$

Rod EF. The rod is in compression (Fig. 1), where the magnitude of the force is P_r and the deformation δ_r:

$$\delta_r = -\frac{P_r L_r}{A_r E_r} = -\frac{P_r(12 \text{ in.})}{\frac{1}{4}\pi(1.5 \text{ in.})^2(10.6 \times 10^6 \text{ psi})} = -0.6406 \times 10^{-6} P_r \quad \textbf{(2)}$$

Displacement of D Relative to B. Tightening the nuts one-quarter of a turn causes ends D and H of the bolts to undergo a displacement of $\frac{1}{4}$(0.1 in.) *relative to casting B*. Considering end D,

$$\delta_{D/B} = \tfrac{1}{4}(0.1 \text{ in.}) = 0.025 \text{ in.} \quad \textbf{(3)}$$

But $\delta_{D/B} = \delta_D - \delta_B$, where δ_D and δ_B represent the displacements of D and B. If casting A is held in a fixed position while the nuts at D and H are being tightened, these displacements are equal to the deformations of the bolts and of the rod, respectively. Therefore,

$$\delta_{D/B} = \delta_b - \delta_r \quad \textbf{(4)}$$

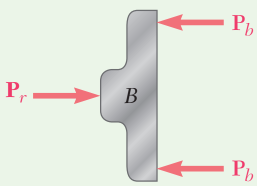

Fig. 2 Free-body diagram of rigid casting.

Substituting from Eqs. (1), (2), and (3) into Eq. (4),

$$0.025 \text{ in.} = 1.405 \times 10^{-6} P_b + 0.6406 \times 10^{-6} P_r \quad \textbf{(5)}$$

Free Body: Casting B (Fig. 2)

$$\overset{+}{\rightarrow} \Sigma F = 0: \qquad P_r - 2P_b = 0 \qquad P_r = 2P_b \quad \textbf{(6)}$$

Forces in Bolts and Rod Substituting for P_r from Eq. (6) into Eq. (5), we have

$$0.025 \text{ in.} = 1.405 \times 10^{-6} P_b + 0.6406 \times 10^{-6}(2P_b)$$

$$P_b = 9.307 \times 10^3 \text{ lb} = 9.307 \text{ kips}$$

$$P_r = 2P_b = 2(9.307 \text{ kips}) = 18.61 \text{ kips}$$

Stress in Rod

$$\sigma_r = \frac{P_r}{A_r} = \frac{18.61 \text{ kips}}{\frac{1}{4}\pi(1.5 \text{ in.})^2} \qquad \boldsymbol{\sigma_r = 10.53 \text{ ksi}} \quad ◀$$

REFLECT and THINK: This is an example of a *statically indeterminate* problem, where the determination of the member forces could not be found by equilibrium alone. By considering the relative displacement characteristics of the members, you can obtain additional equations necessary to solve such problems. Situations like this will be examined in more detail in the following section.

Problems

2.1 A nylon thread is subjected to a 8.5-N tension force. Knowing that $E = 3.3$ GPa and that the length of the thread increases by 1.1%, determine (*a*) the diameter of the thread, (*b*) the stress in the thread.

2.2 A 4.8-ft-long steel wire of $\frac{1}{4}$ -in.-diameter is subjected to a 750-lb tensile load. Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the elongation of the wire, (*b*) the corresponding normal stress.

2.3 An 18-m-long steel wire of 5-mm diameter is to be used in the manufacture of a prestressed concrete beam. It is observed that the wire stretches 45 mm when a tensile force **P** is applied. Knowing that $E = 200$ GPa, determine (*a*) the magnitude of the force **P**, (*b*) the corresponding normal stress in the wire.

2.4 Two gage marks are placed exactly 250 mm apart on a 12-mm-diameter aluminum rod with $E = 73$ GPa and an ultimate strength of 140 MPa. Knowing that the distance between the gage marks is 250.28 mm after a load is applied, determine (*a*) the stress in the rod, (*b*) the factor of safety.

2.5 An aluminum pipe must not stretch more than 0.05 in. when it is subjected to a tensile load. Knowing that $E = 10.1 \times 10^6$ psi and that the maximum allowable normal stress is 14 ksi, determine (*a*) the maximum allowable length of the pipe, (*b*) the required area of the pipe if the tensile load is 127.5 kips.

2.6 A control rod made of yellow brass must not stretch more than 3 mm when the tension in the wire is 4 kN. Knowing that $E = 105$ GPa and that the maximum allowable normal stress is 180 MPa, determine (*a*) the smallest diameter rod that should be used, (*b*) the corresponding maximum length of the rod.

2.7 A steel control rod is 5.5 ft long and must not stretch more than 0.04 in. when a 2-kip tensile load is applied to it. Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the smallest diameter rod that should be used, (*b*) the corresponding normal stress caused by the load.

2.8 A cast-iron tube is used to support a compressive load. Knowing that $E = 10 \times 10^6$ psi and that the maximum allowable change in length is 0.025%, determine (*a*) the maximum normal stress in the tube, (*b*) the minimum wall thickness for a load of 1600 lb if the outside diameter of the tube is 2.0 in.

2.9 A 4-m-long steel rod must not stretch more than 3 mm and the normal stress must not exceed 150 MPa when the rod is subjected to a 10-kN axial load. Knowing that $E = 200$ GPa, determine the required diameter of the rod.

$E = 3.2$ GPa, that the maximum allowable normal stress is 40 MPa, and that the length of the thread must not increase by more than 1%, determine the required diameter of the thread.

2.11 A block of 10-in. length and 1.8×1.6-in. cross section is to support a centric compressive load **P**. The material to be used is a bronze for which $E = 14 \times 10^6$ psi. Determine the largest load that can be applied, knowing that the normal stress must not exceed 18 ksi and that the decrease in length of the block should be at most 0.12% of its original length.

2.12 A square yellow-brass bar must not stretch more than 2.5 mm when it is subjected to a tensile load. Knowing that $E = 105$ GPa and that the allowable tensile strength is 180 MPa, determine (*a*) the maximum allowable length of the bar, (*b*) the required dimensions of the cross section if the tensile load is 40 kN.

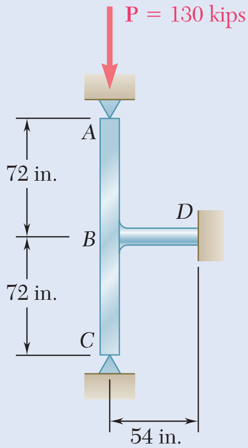

Fig. P2.13

2.13 Rod *BD* is made of steel ($E = 29 \times 10^6$ psi) and is used to brace the axially compressed member *ABC*. The maximum force that can be developed in member *BD* is $0.02P$. If the stress must not exceed 18 ksi and the maximum change in length of *BD* must not exceed 0.001 times the length of *ABC*, determine the smallest-diameter rod that can be used for member *BD*.

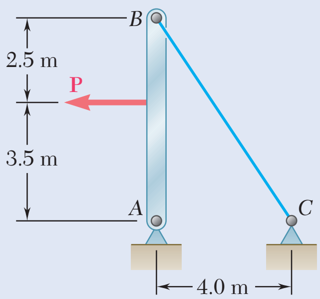

Fig. P2.14

2.14 The 4-mm-diameter cable *BC* is made of a steel with $E = 200$ GPa. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load **P** that can be applied as shown.

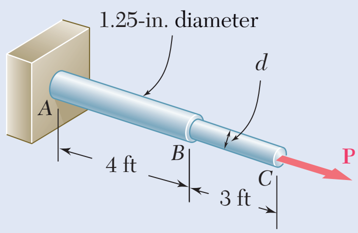

Fig. P2.15

2.15 A single axial load of magnitude $P = 15$ kips is applied at end *C* of the steel rod *ABC*. Knowing that $E = 30 \times 10^6$ psi, determine the diameter *d* of portion *BC* for which the deflection of point *C* will be 0.05 in.

2.16 A 250-mm-long aluminum tube ($E = 70$ GPa) of 36-mm outer diameter and 28-mm inner diameter can be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod ($E = 105$ GPa) of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (*a*) the average normal stress in the tube and in the rod, (*b*) the deformations of the tube and of the rod.

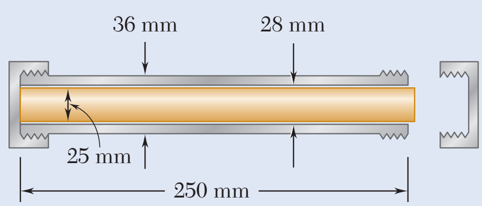

Fig. *P2.16*

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vinyl ($E = 0.45 \times 10^6$ psi) and is subjected to a 350-lb tensile load. Determine (*a*) the total deformation of the specimen, (*b*) the deformation of its central portion *BC*.

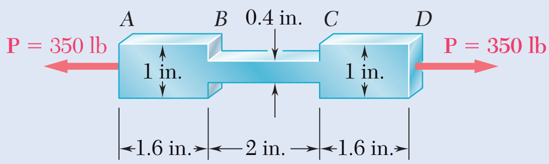


Fig. P2.17

2.18 The brass tube *AB* (E = 105 GPa) has a cross-sectional area of 140 mm^2 and is fitted with a plug at *A*. The tube is attached at *B* to a rigid plate that is itself attached at *C* to the bottom of an aluminum cylinder (E = 72 GPa) with a cross-sectional area of 250 mm^2. The cylinder is then hung from a support at *D*. In order to close the cylinder, the plug must move down through 1 mm. Determine the force **P** that must be applied to the cylinder.

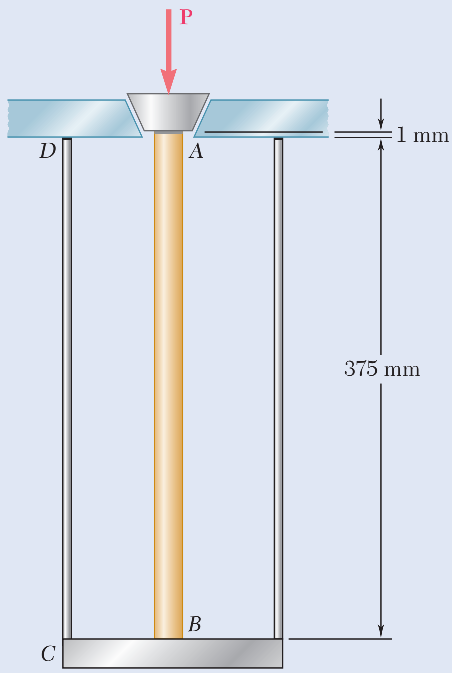


Fig. *P2.18*

2.19 Both portions of the rod *ABC* are made of an aluminum for which E = 70 GPa. Knowing that the magnitude of **P** is 4 kN, determine (*a*) the value of **Q** so that the deflection at *A* is zero, (*b*) the corresponding deflection of *B*.

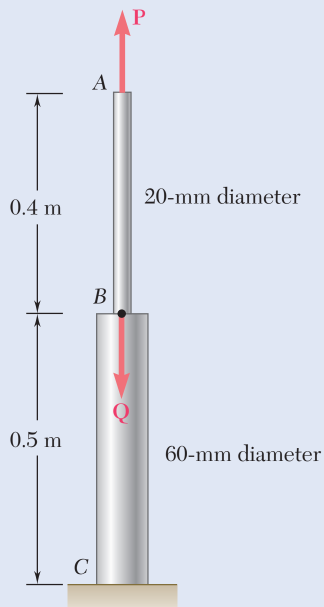


Fig. P2.19 and P2.20

2.20 The rod *ABC* is made of an aluminum for which E = 70 GPa. Knowing that P = 6 kN and Q = 42 kN, determine the deflection of (*a*) point *A*, (*b*) point *B*.

2.21 For the steel truss (E = 200 GPa) and loading shown, determine the deformations of members *AB* and *AD*, knowing that their cross-sectional areas are 2400 mm^2 and 1800 mm^2, respectively.

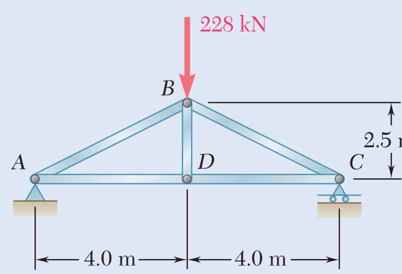


Fig. P2.21

determine the deformations of members BD and DE, knowing that their cross-sectional areas are 2 in^2 and 3 in^2, respectively.

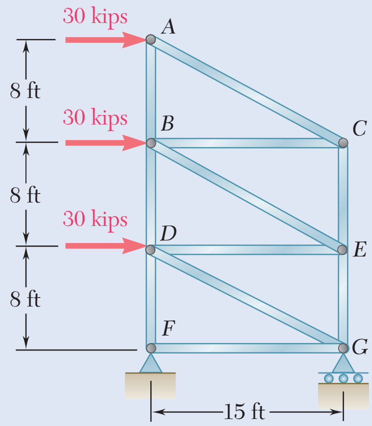


Fig. ***P2.22***

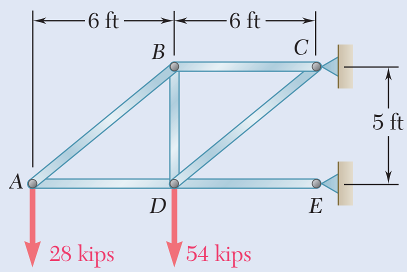


Fig. P2.23

2.23 Members AB and BC are made of steel ($E = 29 \times 10^6$ psi) with cross-sectional areas of 0.80 in^2 and 0.64 in^2, respectively. For the loading shown, determine the elongation of (*a*) member AB, (*b*) member BC.

2.24 The steel frame ($E = 200$ GPa) shown has a diagonal brace BD with an area of 1920 mm^2. Determine the largest allowable load **P** if the change in length of member BD is not to exceed 1.6 mm.

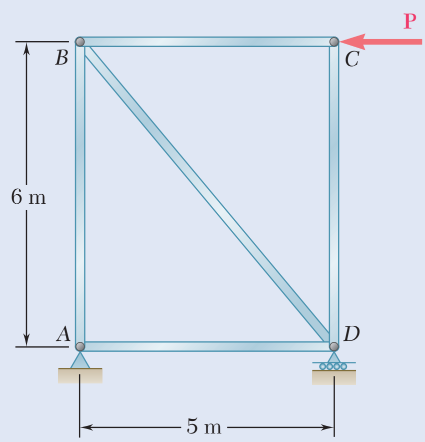


Fig. P2.24

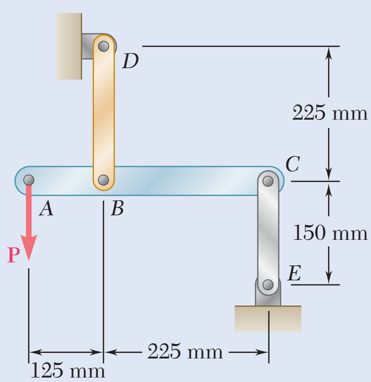


Fig. P2.25

2.25 Link BD is made of brass ($E = 105$ GPa) and has a cross-sectional area of 240 mm^2. Link CE is made of aluminum ($E = 72$ GPa) and has a cross-sectional area of 300 mm^2. Knowing that they support rigid member ABC, determine the maximum force **P** that can be applied vertically at point A if the deflection of A is not to exceed 0.35 mm.

Deflection δ_T. Because of a temperature rise of $50° - 20° = 30°C$, the length of the brass cylinder increases by δ_T. (Fig. 2*a*).

$$\delta_T = L(\Delta T)\alpha = (0.3 \text{ m})(30°C)(20.9 \times 10^{-6}/°C) = 188.1 \times 10^{-6} \text{ m} \downarrow$$

Deflection δ_1. From Fig. 2*b*, note that $\delta_D = 0.4\delta_C$ and $\delta_1 = \delta_D + \delta_{B/D}$.

$$\delta_C = \frac{R_A L}{AE} = \frac{R_A(0.9 \text{ m})}{\frac{1}{4}\pi(0.022 \text{ m})^2(200 \text{ GPa})} = 11.84 \times 10^{-9} R_A \uparrow$$

$$\delta_D = 0.40\delta_C = 0.4(11.84 \times 10^{-9} R_A) = 4.74 \times 10^{-9} R_A \uparrow$$

$$\delta_{B/D} = \frac{R_B L}{AE} = \frac{R_B(0.3 \text{ m})}{\frac{1}{4}\pi(0.03 \text{ m})^2(105 \text{ GPa})} = 4.04 \times 10^{-9} R_B \uparrow$$

Recall from Eq. (1) that $R_A = 0.4R_B$, so

$$\delta_1 = \delta_D + \delta_{B/D} = [4.74(0.4R_B) + 4.04R_B]10^{-9} = 5.94 \times 10^{-9} R_B \uparrow$$

But $\delta_T = \delta_1$: $\qquad 188.1 \times 10^{-6} \text{ m} = 5.94 \times 10^{-9} R_B \qquad R_B = 31.7 \text{ kN}$

Stress in Cylinder: $\sigma_B = \dfrac{R_B}{A} = \dfrac{31.7 \text{ kN}}{\frac{1}{4}\pi(0.03 \text{ m})^2} \qquad \sigma_B = 44.8 \text{ MPa}$ ◀

REFLECT and THINK: This example illustrates the large stresses that can develop in statically indeterminate systems due to even modest temperature changes. Note that if this assembly was statically determinate (i.e., the steel rod was removed), no stress at all would develop in the cylinder due to the temperature change.

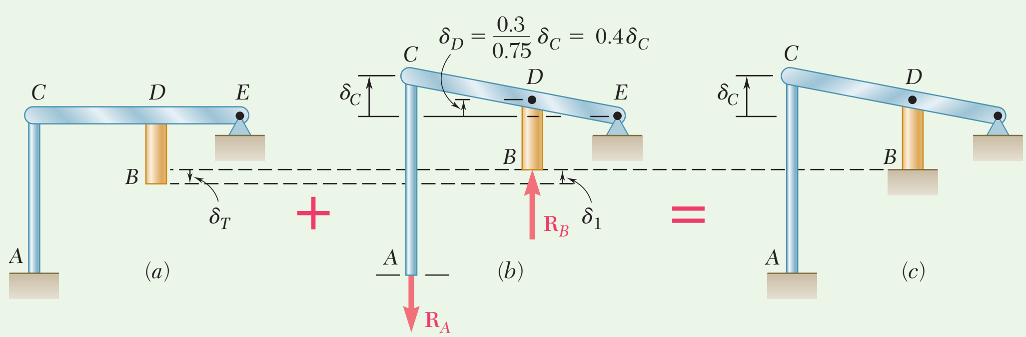


Fig. 2 Superposition of thermal and restraint force deformations (*a*) Support at *B* removed. (*b*) Reaction at *B* applied. (*c*) Final position.

Problems

2.33 An axial centric force of magnitude $P = 450$ kN is applied to the composite block shown by means of a rigid end plate. Knowing that $h = 10$ mm, determine the normal stress in (*a*) the brass core, (*b*) the aluminum plates.

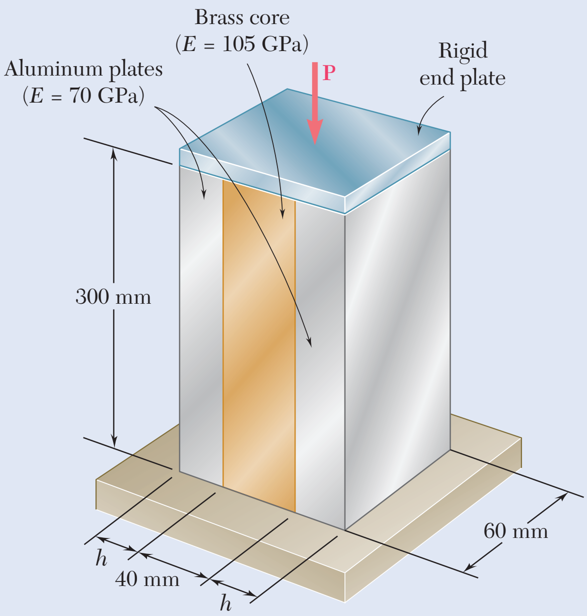


Fig. P2.33

2.34 For the composite block shown in Prob. 2.33, determine (*a*) the value of h if the portion of the load carried by the aluminum plates is half the portion of the load carried by the brass core, (*b*) the total load if the stress in the brass is 80 MPa.

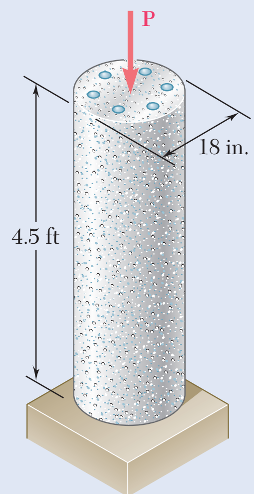


Fig. P2.35

2.35 The 4.5-ft concrete post is reinforced with six steel bars, each with a $1\frac{1}{8}$-in. diameter. Knowing that $E_s = 29 \times 10^6$ psi and $E_c = 4.2 \times 10^6$ psi, determine the normal stresses in the steel and in the concrete when a 350-kip axial centric force **P** is applied to the post.

2.36 For the post of Prob. 2.35, determine the maximum centric force that can be applied if the allowable normal stress is 20 ksi in the steel and 2.4 ksi in the concrete.

2.37 An axial force of 200 kN is applied to the assembly shown by means of rigid end plates. Determine (*a*) the normal stress in the aluminum shell, (*b*) the corresponding deformation of the assembly.

2.38 The length of the assembly shown decreases by 0.40 mm when an axial force is applied by means of rigid end plates. Determine (*a*) the magnitude of the applied force, (*b*) the corresponding stress in the brass core.

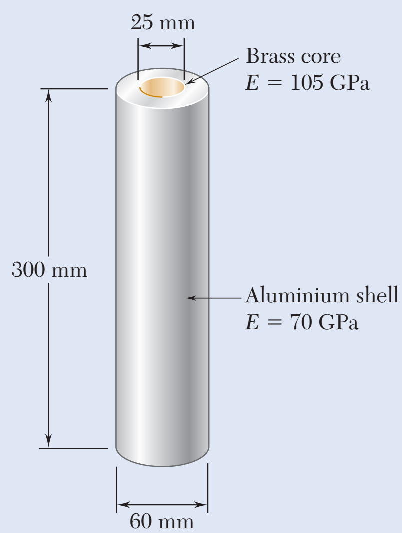


Fig. *P2.37* and *P2.38*

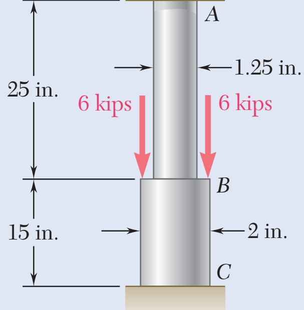


Fig. P2.39

BC is restrained at both ends and supports two 6-kip loads as shown. Knowing that $E = 0.45 \times 10^6$ psi, determine (*a*) the reactions at A and C, (*b*) the normal stress in each portion of the rod.

2.40 Three steel rods ($E = 29 \times 10^6$ psi) support an 8.5-kip load **P**. Each of the rods AB and CD has a 0.32-in^2 cross-sectional area and rod EF has a 1-in^2 cross-sectional area. Neglecting the deformation of bar BED, determine (*a*) the change in length of rod EF, (*b*) the stress in each rod.

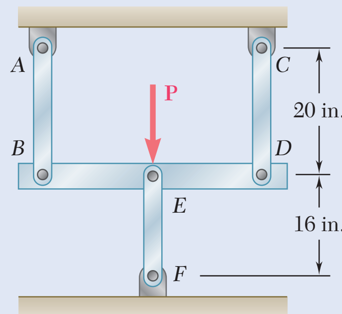


Fig. ***P2.40***

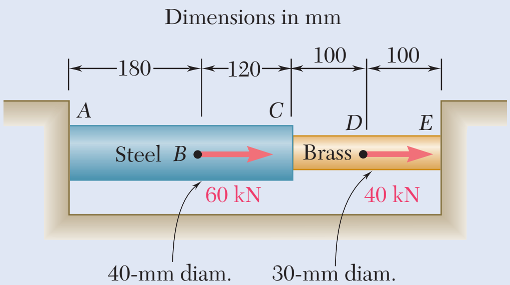


Fig. P2.41

2.41 Two cylindrical rods, one of steel and the other of brass, are joined at C and restrained by rigid supports at A and E. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine (*a*) the reactions at A and E, (*b*) the deflection of point C.

2.42 Solve Prob. 2.41, assuming that rod AC is made of brass and rod CE is made of steel.

2.43 Each of the rods BD and CE is made of brass ($E = 105$ GPa) and has a cross-sectional area of 200 mm^2. Determine the deflection of end A of the rigid member ABC caused by the 2-kN load.

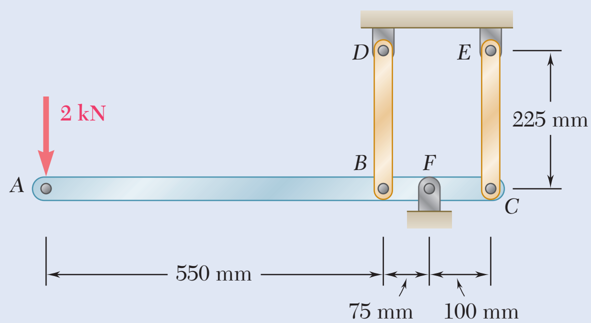


Fig. P2.43

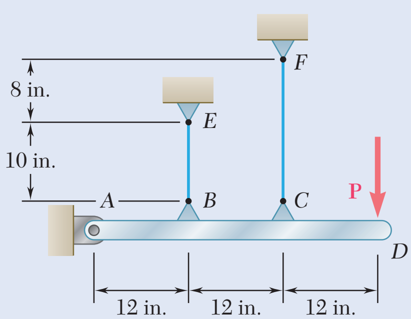


Fig. P2.44

2.44 The rigid bar AD is supported by two steel wires of $\frac{1}{16}$-in. diameter ($E = 29 \times 10^6$ psi) and a pin and bracket at A. Knowing that the wires were initially taut, determine (*a*) the additional tension in each wire when a 220-lb load **P** is applied at D, (*b*) the corresponding deflection of point D.

material. The cross-sectional area of the wire at B is equal to half of the cross-sectional area of the wires at A and C. Determine the tension in each wire caused by the load **P** shown.

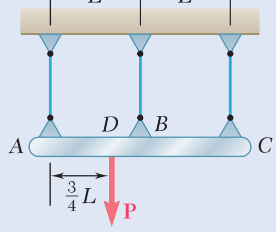


Fig. P2.45

2.46 The rigid bar AD is supported by two steel wires of $\frac{1}{16}$-in. diameter ($E = 29 \times 10^6$ psi) and a pin and bracket at D. Knowing that the wires were initially taut, determine (*a*) the additional tension in each wire when a 120-lb load **P** is applied at B, (*b*) the corresponding deflection of point B.

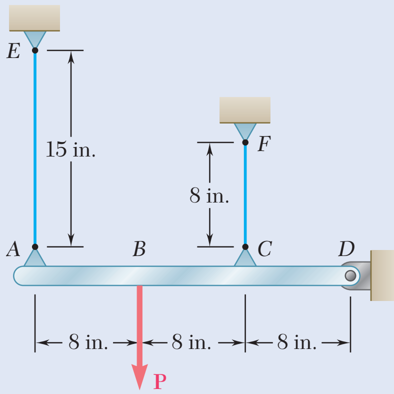


Fig. *P2.46*

2.47 The aluminum shell is fully bonded to the brass core and the assembly is unstressed at a temperature of 15°C. Considering only axial deformations, determine the stress in the aluminum when the temperature reaches 195°C.

2.48 Solve Prob. 2.47, assuming that the core is made of steel ($E_s = 200$ GPa, $\alpha_s = 11.7 \times 10^{-6}$/°C) instead of brass.

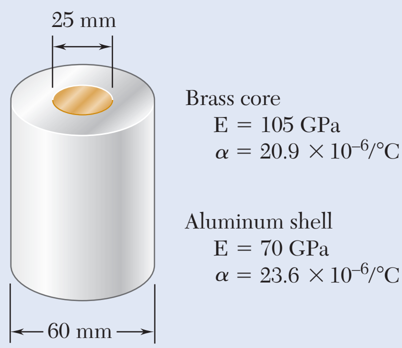


Fig. P2.47

2.49 The brass shell ($\alpha_b = 11.6 \times 10^{-6}$/°F) is fully bonded to the steel core ($\alpha_s = 6.5 \times 10^{-6}$/°F). Determine the largest allowable increase in temperature if the stress in the steel core is not to exceed 8 ksi.

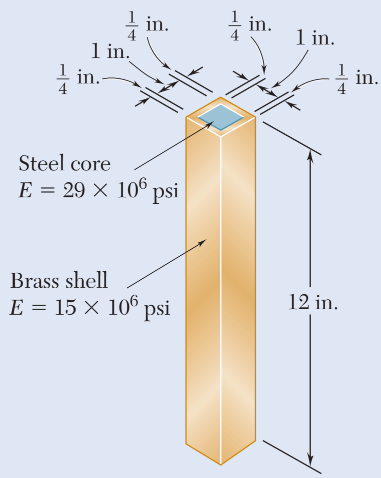


Fig. *P2.49*

reinforced with six steel bars, each of $\frac{7}{8}$-in. diameter ($E_s = 29 \times 10^6$ psi and $\alpha_s = 6.5 \times 10^{-6}/°\text{F}$). Determine the normal stresses induced in the steel and in the concrete by a temperature rise of 65°F.

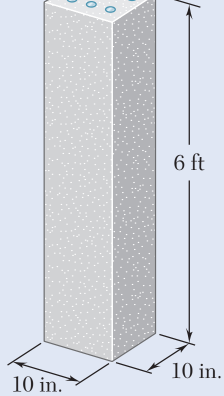


Fig. P2.50

2.51 A rod consisting of two cylindrical portions AB and BC is restrained at both ends. Portion AB is made of steel ($E_s = 200$ GPa, $\alpha_s = 11.7 \times 10^{-6}/°\text{C}$) and portion BC is made of brass ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}/°\text{C}$). Knowing that the rod is initially unstressed, determine the compressive force induced in ABC when there is a temperature rise of 50°C.

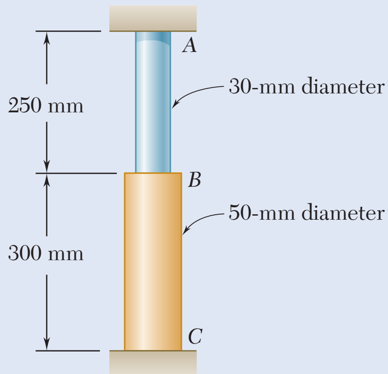


Fig. P2.51

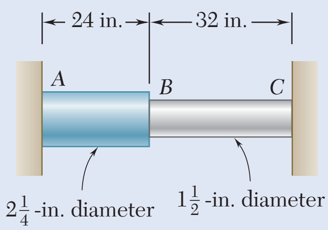


Fig. P2.52

2.52 A rod consisting of two cylindrical portions AB and BC is restrained at both ends. Portion AB is made of steel ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}/°\text{F}$) and portion BC is made of aluminum ($E_a = 10.4 \times 10^6$ psi, $\alpha_a = 13.3 \times 10^{-6}/°\text{F}$). Knowing that the rod is initially unstressed, determine (*a*) the normal stresses induced in portions AB and BC by a temperature rise of 70°F, (*b*) the corresponding deflection of point B.

2.53 Solve Prob. 2.52, assuming that portion AB of the composite rod is made of aluminum and portion BC is made of steel.

2.54 The steel rails of a railroad track ($E_s = 200$ GPa, $\alpha_s = 11.7 \times 10^{-6}/°\text{C}$) were laid at a temperature of 6°C. Determine the normal stress in the rails when the temperature reaches 48°C, assuming that the rails (*a*) are welded to form a continuous track, (*b*) are 10 m long with 3-mm gaps between them.

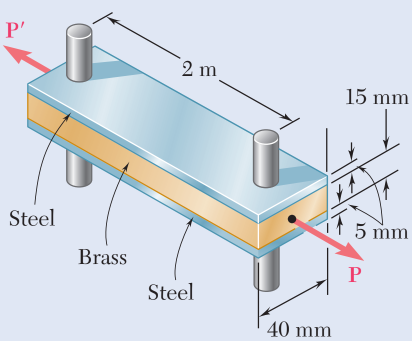
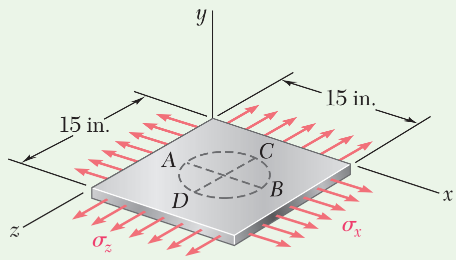


Fig. P2.55

2.55 Two steel bars ($E_s = 200$ GPa and $\alpha_s = 11.7 \times 10^{-6}/°\text{C}$) are used to reinforce a brass bar ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}/°\text{C}$) that is subjected to a load $P = 25$ kN. When the steel bars were fabricated, the distance between the centers of the holes that were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (*a*) the increase in temperature that was required to fit the steel bars on the pins, (*b*) the stress in the brass bar after the load is applied to it.



Sample Problem 2.5

A circle of diameter $d = 9$ in. is scribed on an unstressed aluminum plate of thickness $t = \frac{3}{4}$ in. Forces acting in the plane of the plate later cause normal stresses $\sigma_x = 12$ ksi and $\sigma_z = 20$ ksi. For $E = 10 \times 10^6$ psi and $\nu = \frac{1}{3}$, determine the change in (*a*) the length of diameter *AB*, (*b*) the length of diameter *CD*, (*c*) the thickness of the plate, and (*d*) the volume of the plate.

STRATEGY: You can use the generalized Hooke's Law to determine the components of strain. These strains can then be used to evaluate the various dimensional changes to the plate, and through the dilatation, also assess the volume change.

ANALYSIS:

Hooke's Law. Note that $\sigma_y = 0$. Using Eqs. (2.20), find the strain in each of the coordinate directions.

$$\epsilon_x = +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$= \frac{1}{10 \times 10^6 \text{ psi}}\left[(12 \text{ ksi}) - 0 - \frac{1}{3}(20 \text{ ksi})\right] = +0.533 \times 10^{-3} \text{ in./in.}$$

$$\epsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$= \frac{1}{10 \times 10^6 \text{ psi}}\left[-\frac{1}{3}(12 \text{ ksi}) + 0 - \frac{1}{3}(20 \text{ ksi})\right] = -1.067 \times 10^{-3} \text{ in./in.}$$

$$\epsilon_z = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$= \frac{1}{10 \times 10^6 \text{ psi}}\left[-\frac{1}{3}(12 \text{ ksi}) - 0 + (20 \text{ ksi})\right] = +1.600 \times 10^{-3} \text{ in./in.}$$

a. Diameter *AB*. The change in length is $\delta_{B/A} = \epsilon_x d$.

$$\delta_{B/A} = \epsilon_x d = (+0.533 \times 10^{-3} \text{ in./in.})(9 \text{ in.})$$

$$\delta_{B/A} = +4.8 \times 10^{-3} \text{ in.} \quad \blacktriangleleft$$

b. Diameter *CD*.

$$\delta_{C/D} = \epsilon_z d = (+1.600 \times 10^{-3} \text{ in./in.})(9 \text{ in.})$$

$$\delta_{C/D} = +14.4 \times 10^{-3} \text{ in.} \quad \blacktriangleleft$$

c. Thickness. Recalling that $t = \frac{3}{4}$ in.,

$$\delta_t = \epsilon_y t = (-1.067 \times 10^{-3} \text{ in./in.})(\tfrac{3}{4} \text{ in.})$$

$$\delta_t = -0.800 \times 10^{-3} \text{ in.} \quad \blacktriangleleft$$

d. Volume of the Plate. Using Eq. (2.21),

$$e = \epsilon_x + \epsilon_y + \epsilon_z = (+0.533 - 1.067 + 1.600)10^{-3} = +1.067 \times 10^{-3}$$

$$\Delta V = eV = +1.067 \times 10^{-3}[(15 \text{ in.})(15 \text{ in.})(\tfrac{3}{4} \text{ in.})]$$

$$\Delta V = +0.180 \text{ in}^3 \quad \blacktriangleleft$$

Problems

2.61 A standard tension test is used to determine the properties of an experimental plastic. The test specimen is a $\frac{5}{8}$-in.-diameter rod and it is subjected to an 800-lb tensile force. Knowing that an elongation of 0.45 in. and a decrease in diameter of 0.025 in. are observed in a 5-in. gage length, determine the modulus of elasticity, the modulus of rigidity, and Poisson's ratio for the material.

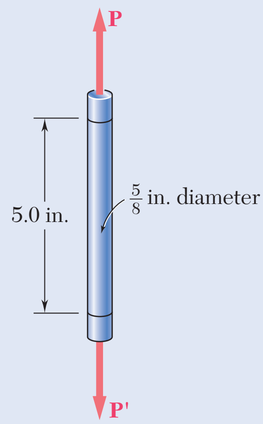


Fig. P2.61

2.62 A 2-m length of an aluminum pipe of 240-mm outer diameter and 10-mm wall thickness is used as a short column to carry a 640-kN centric axial load. Knowing that $E = 73$ GPa and $\nu = 0.33$, determine (*a*) the change in length of the pipe, (*b*) the change in its outer diameter, (*c*) the change in its wall thickness.

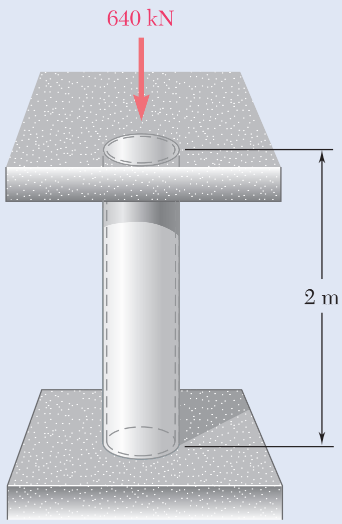


Fig. *P2.62*

2.63 A line of slope 4:10 has been scribed on a cold-rolled yellow-brass plate, 150 mm wide and 6 mm thick. Knowing that $E = 105$ GPa and $\nu = 0.34$, determine the slope of the line when the plate is subjected to a 200-kN centric axial load as shown.

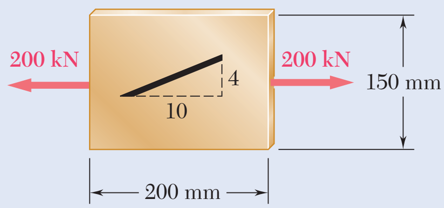


Fig. P2.63

2.64 A 2.75-kN tensile load is applied to a test coupon made from 1.6-mm flat steel plate ($E = 200$ GPa, $\nu = 0.30$). Determine the resulting change (*a*) in the 50-mm gage length, (*b*) in the width of portion *AB* of the test coupon, (*c*) in the thickness of portion *AB*, (*d*) in the cross-sectional area of portion *AB*.

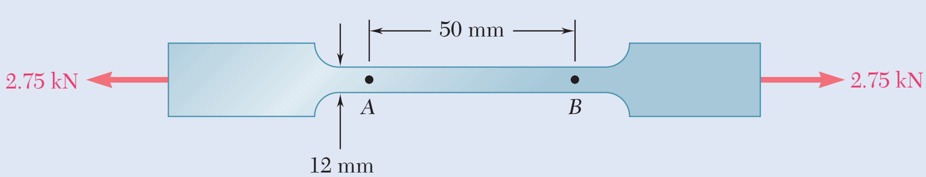


Fig. P2.64

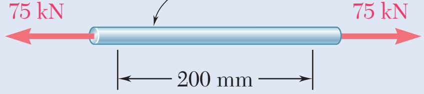


Fig. P2.65

subjected to a tension force of 75 kN. Knowing that $\nu = 0.30$ and $E = 200$ GPa, determine (*a*) the elongation of the rod in a 200-mm gage length, (*b*) the change in diameter of the rod.

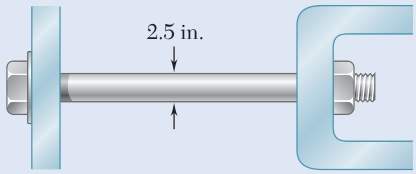


Fig. P2.66

2.66 The change in diameter of a large steel bolt is carefully measured as the nut is tightened. Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine the internal force in the bolt if the diameter is observed to decrease by 0.5×10^{-3} in.

2.67 The brass rod *AD* is fitted with a jacket that is used to apply a hydrostatic pressure of 48 MPa to the 240-mm portion *BC* of the rod. Knowing that $E = 105$ GPa and $\nu = 0.33$, determine (*a*) the change in the total length *AD*, (*b*) the change in diameter at the middle of the rod.

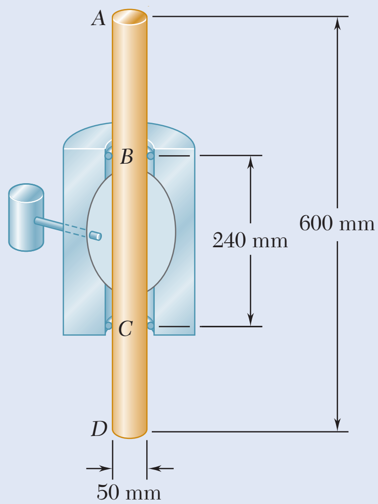


Fig. P2.67

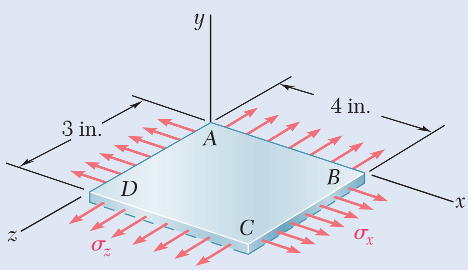


Fig. P2.68

2.68 A fabric used in air-inflated structures is subjected to a biaxial loading that results in normal stresses $\sigma_x = 18$ ksi and $\sigma_z = 24$ ksi. Knowing that the properties of the fabric can be approximated as $E = 12.6 \times 10^6$ psi and $\nu = 0.34$, determine the change in length of (*a*) side *AB*, (*b*) side *BC*, (*c*) diagonal *AC*.

2.69 A 1-in. square was scribed on the side of a large steel pressure vessel. After pressurization the biaxial stress condition at the square is as shown. Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine the change in length of (*a*) side *AB*, (*b*) side *BC*, (*c*) diagonal *AC*.

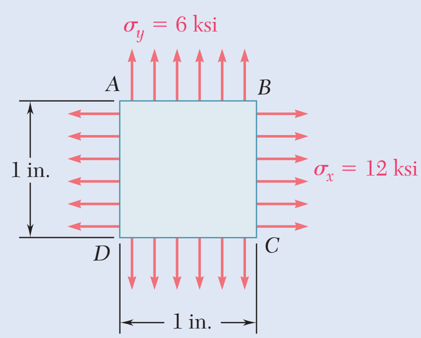


Fig. P2.69

$E = 45$ GPa and $\nu = 0.35$. Knowing that $\sigma_x = -180$ MPa, determine (*a*) the magnitude of σ_y for which the change in the height of the block will be zero, (*b*) the corresponding change in the area of the face *ABCD*, (*c*) the corresponding change in the volume of the block.

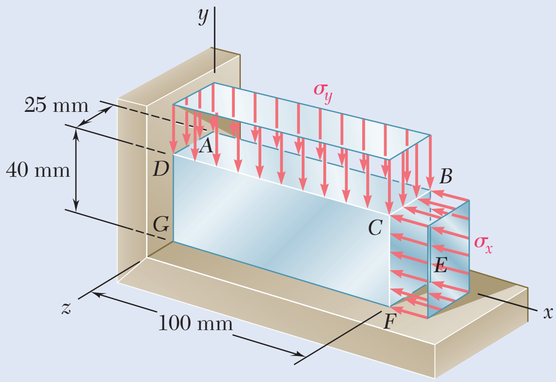


Fig. P2.70

2.71 The homogeneous plate *ABCD* is subjected to a biaxial loading as shown. It is known that $\sigma_z = \sigma_0$ and that the change in length of the plate in the *x* direction must be zero, that is, $\epsilon_x = 0$. Denoting by *E* the modulus of elasticity and by ν Poisson's ratio, determine (*a*) the required magnitude of σ_x, (*b*) the ratio σ_0/ϵ_z.

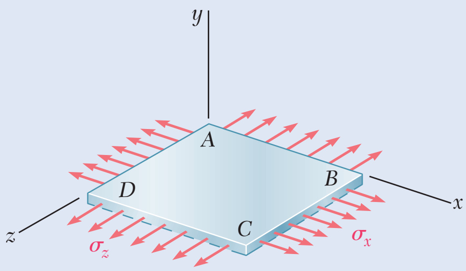


Fig. *P2.71*

2.72 For a member under axial loading, express the normal strain ϵ' in a direction forming an angle of 45° with the axis of the load in terms of the axial strain ϵ_x by (*a*) comparing the hypotenuses of the triangles shown in Fig. 2.43, which represent respectively an element before and after deformation, (*b*) using the values of the corresponding stresses σ' and σ_x shown in Fig. 1.38, and the generalized Hooke's law.

2.73 In many situations it is known that the normal stress in a given direction is zero. For example, $\sigma_z = 0$ in the case of the thin plate shown. For this case, which is known as *plane stress*, show that if the strains ϵ_x and ϵ_y have been determined experimentally, we can express σ_x, σ_y, and ϵ_z as follows:

$$\sigma_x = E\frac{\epsilon_x + \nu\epsilon_y}{1 - \nu^2}$$

$$\sigma_y = E\frac{\epsilon_y + \nu\epsilon_x}{1 - \nu^2}$$

$$\epsilon_z = -\frac{\nu}{1 - \nu}(\epsilon_x + \epsilon_y)$$

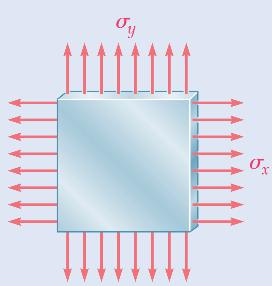


Fig. P2.73

occurring in a given direction. For example, $\epsilon_z = 0$ in the case shown, where longitudinal movement of the long prism is prevented at every point. Plane sections perpendicular to the longitudinal axis remain plane and the same distance apart. Show that for this situation, which is known as *plane strain*, we can express σ_z, ϵ_x, and ϵ_y as follows:

$$\sigma_z = \nu(\sigma_x + \sigma_y)$$
$$\epsilon_x = \frac{1}{E}[(1 - \nu^2)\sigma_x - \nu(1 + \nu)\sigma_y]$$
$$\epsilon_y = \frac{1}{E}[(1 - \nu^2)\sigma_y - \nu(1 + \nu)\sigma_x]$$

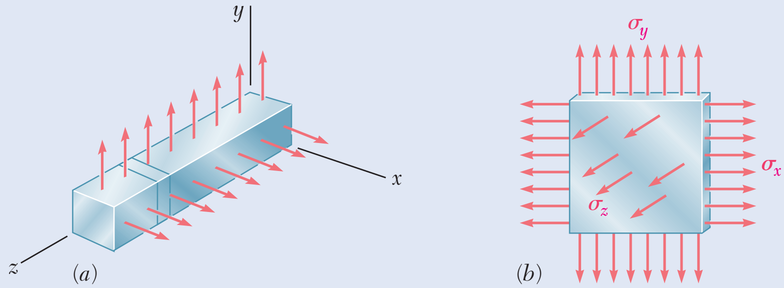


Fig. P2.74

2.75 The plastic block shown is bonded to a rigid support and to a vertical plate to which a 55-kip load **P** is applied. Knowing that for the plastic used $G = 150$ ksi, determine the deflection of the plate.

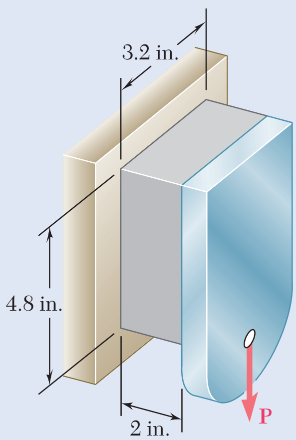


Fig. *P2.75*

2.76 What load **P** should be applied to the plate of Prob. 2.75 to produce a $\frac{1}{16}$-in. deflection?

2.77 Two blocks of rubber with a modulus of rigidity $G = 12$ MPa are bonded to rigid supports and to a plate AB. Knowing that $c = 100$ mm and $P = 45$ kN, determine the smallest allowable dimensions a and b of the blocks if the shearing stress in the rubber is not to exceed 1.4 MPa and the deflection of the plate is to be at least 5 mm.

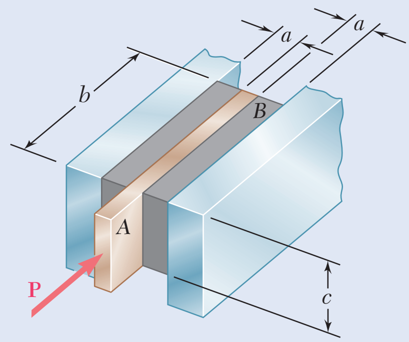


Fig. P2.77 and P2.78

2.78 Two blocks of rubber with a modulus of rigidity $G = 10$ MPa are bonded to rigid supports and to a plate AB. Knowing that $b = 200$ mm and $c = 125$ mm, determine the largest allowable load P and the smallest allowable thickness a of the blocks if the shearing stress in the rubber is not to exceed 1.5 MPa and the deflection of the plate is to be at least 6 mm.

girder as shown to provide flexibility during earthquakes. The beam must not displace more than $\frac{3}{8}$ in. when a 5-kip lateral load is applied as shown. Knowing that the maximum allowable shearing stress is 60 psi, determine (*a*) the smallest allowable dimension *b*, (*b*) the smallest required thickness *a*.

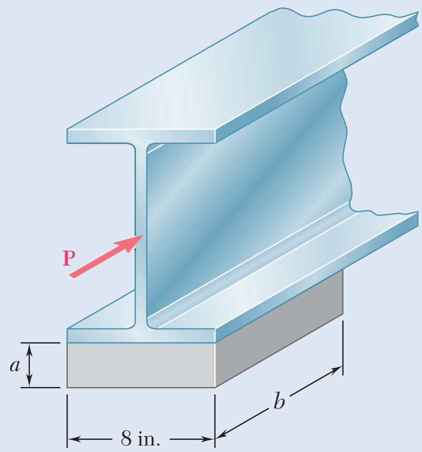


Fig. P2.79

2.80 For the elastomeric bearing in Prob. 2.79 with $b = 10$ in. and $a = 1$ in., determine the shearing modulus G and the shear stress τ for a maximum lateral load $P = 5$ kips and a maximum displacement $\delta = 0.4$ in.

2.81 A vibration isolation unit consists of two blocks of hard rubber bonded to a plate *AB* and to rigid supports as shown. Knowing that a force of magnitude $P = 25$ kN causes a deflection $\delta = 1.5$ mm of plate *AB*, determine the modulus of rigidity of the rubber used.

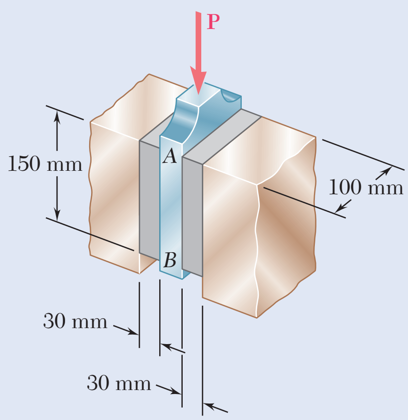


Fig. P2.81 and P2.82

2.82 A vibration isolation unit consists of two blocks of hard rubber with a modulus of rigidity $G = 19$ MPa bonded to a plate *AB* and to rigid supports as shown. Denoting by P the magnitude of the force applied to the plate and by δ the corresponding deflection, determine the effective spring constant, $k = P/\delta$, of the system.

Since $P_{CE} = P_{AD} = 120$ kN, the stress in rod CE is

$$\sigma_{CE} = \frac{P_{CE}}{A} = \frac{120 \text{ kN}}{500 \text{ mm}^2} = 240 \text{ MPa}$$

The corresponding deflection of point C is

$$\delta_{C_1} = \epsilon L = \frac{\sigma_{CE}}{E} L = \left(\frac{240 \text{ MPa}}{200 \text{ GPa}}\right)(5 \text{ m}) = 6 \text{ mm}$$

The corresponding deflection of point B is

$$\delta_{B_1} = \tfrac{1}{2}(\delta_{A_1} + \delta_{C_1}) = \tfrac{1}{2}(3 \text{ mm} + 6 \text{ mm}) = 4.5 \text{ mm}$$

Since $\delta_B = 10$ mm, plastic deformation will occur.

Plastic Deformation. For $Q = 240$ kN, plastic deformation occurs in rod AD, where $\sigma_{AD} = \sigma_Y = 300$ MPa. Since the stress in rod CE is within the elastic range, δ_C remains equal to 6 mm. From Fig. 3, the deflection δ_A for which $\delta_B = 10$ mm is obtained by writing

$$\delta_{B_2} = 10 \text{ mm} = \tfrac{1}{2}(\delta_{A_2} + 6 \text{ mm}) \qquad \delta_{A_2} = 14 \text{ mm}$$

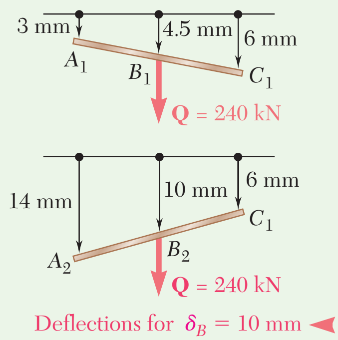


Fig. 3 Deflection of fully-loaded beam.

Unloading. As force $\mathbf{Q}$ is slowly removed, the force P_{AD} decreases along line HJ parallel to the initial portion of the load-deflection diagram of rod AD. The final deflection of point A is

$$\delta_{A_3} = 14 \text{ mm} - 3 \text{ mm} = 11 \text{ mm}$$

Since the stress in rod CE remained within the elastic range, note that the final deflection of point C is zero. Fig. 4 illustrates the final position of the beam.

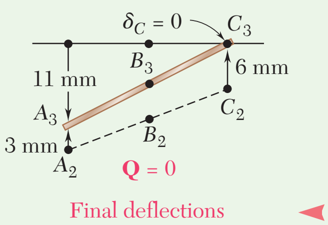


Fig. 4 Beam's final deflections with load removed.

REFLECT and THINK: Due to symmetry in this determinate problem, the axial forces in the rods are equal. Given that the rods have identical material properties and that the cross-sectional area of rod AD is smaller than rod CE, you would therefore expect that rod AD would reach yield first (as assumed in the STRATEGY step).

Problems

2.93 Knowing that, for the plate shown, the allowable stress is 125 MPa, determine the maximum allowable value of P when (*a*) $r = 12$ mm, (*b*) $r = 18$ mm.

2.94 Knowing that $P = 38$ kN, determine the maximum stress when (*a*) $r = 10$ mm, (*b*) $r = 16$ mm, (*c*) $r = 18$ mm.

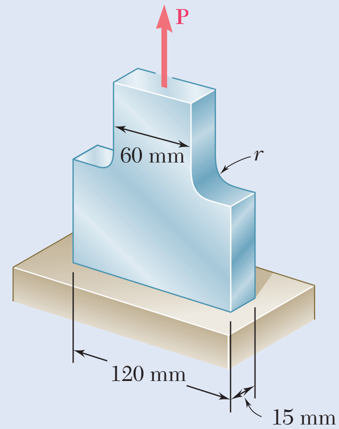


Fig. P2.93 and P2.94

2.95 A hole is to be drilled in the plate at A. The diameters of the bits available to drill the hole range from $\frac{1}{2}$ to $1\frac{1}{2}$ in. in $\frac{1}{4}$-in. increments. If the allowable stress in the plate is 21 ksi, determine (*a*) the diameter d of the largest bit that can be used if the allowable load **P** at the hole is to exceed that at the fillets, (*b*) the corresponding allowable load **P**.

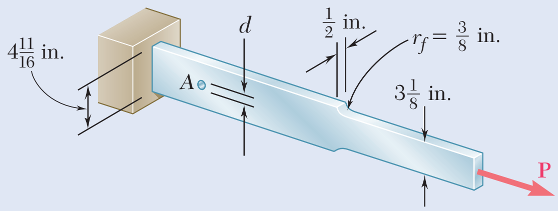


Fig. *P2.95* and *P2.96*

2.96 (*a*) For $P = 13$ kips and $d = \frac{1}{2}$ in., determine the maximum stress in the plate shown. (*b*) Solve part *a*, assuming that the hole at A is not drilled.

2.97 Knowing that the hole has a diameter of 9 mm, determine (*a*) the radius r_f of the fillets for which the same maximum stress occurs at the hole A and at the fillets, (*b*) the corresponding maximum allowable load **P** if the allowable stress is 100 MPa.

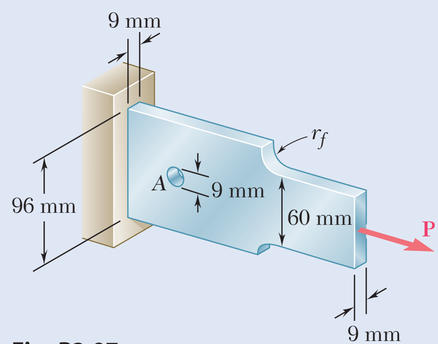


Fig. P2.97

2.98 For $P = 100$ kN, determine the minimum plate thickness t required if the allowable stress is 125 MPa.

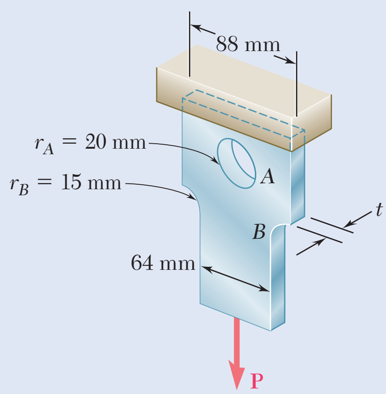


Fig. P2.98

maximum allowable magnitude of the centric load **P**. (*b*) Determine the percent change in the maximum allowable magnitude of **P** if the raised portions are removed at the ends of the specimen.

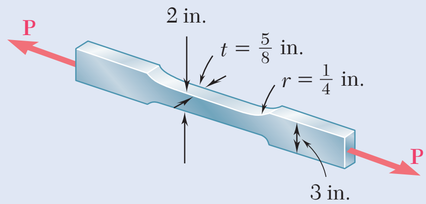


Fig. P2.99

2.100 A centric axial force is applied to the steel bar shown. Knowing that $\sigma_{all} = 20$ ksi, determine the maximum allowable load **P**.

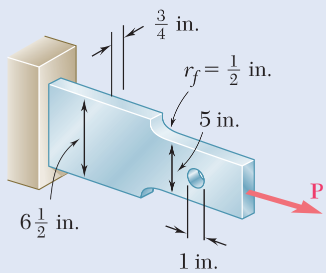


Fig. P2.100

2.101 The cylindrical rod *AB* has a length $L = 5$ ft and a 0.75-in. diameter; it is made of a mild steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 36$ ksi. A force **P** is applied to the bar and then removed to give it a permanent set δ_P. Determine the maximum value of the force **P** and the maximum amount δ_m by which the bar should be stretched if the desired value of δ_P is (*a*) 0.1 in., (*b*) 0.2 in.

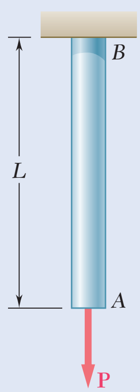


Fig. P2.101 and P2.102

2.102 The cylindrical rod *AB* has a length $L = 6$ ft and a 1.25-in. diameter; it is made of a mild steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 36$ ksi. A force **P** is applied to the bar until end *A* has moved down by an amount δ_m. Determine the maximum value of the force **P** and the permanent set of the bar after the force has been removed, knowing (*a*) $\delta_m = 0.125$ in., (*b*) $\delta_m = 0.250$ in.

with $E = 200$ GPa and $\sigma_Y = 345$ MPa. After the rod has been attached to the rigid lever *CD*, it is found that end *C* is 6 mm too high. A vertical force **Q** is then applied at *C* until this point has moved to position *C*′. Determine the required magnitude of **Q** and the deflection δ_1 if the lever is to *snap* back to a horizontal position after **Q** is removed.

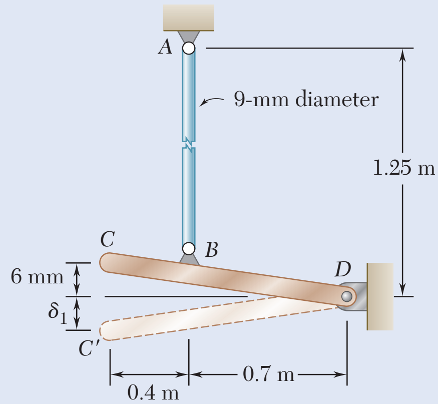


Fig. ***P2.103***

2.104 Solve Prob. 2.103, assuming that the yield point of the mild steel is 250 MPa.

2.105 Rod *ABC* consists of two cylindrical portions *AB* and *BC;* it is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. A force **P** is applied to the rod and then removed to give it a permanent set $\delta_P = 2$ mm. Determine the maximum value of the force **P** and the maximum amount δ_m by which the rod should be stretched to give it the desired permanent set.

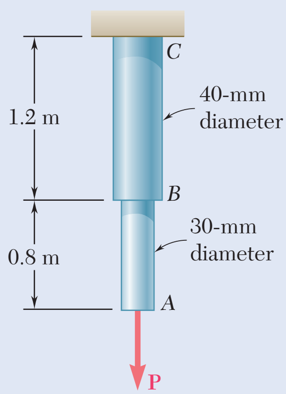


Fig. P2.105 and P2.106

2.106 Rod *ABC* consists of two cylindrical portions *AB* and *BC;* it is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. A force **P** is applied to the rod until its end *A* has moved down by an amount $\delta_m = 5$ mm. Determine the maximum value of the force **P** and the permanent set of the rod after the force has been removed.

a cross-sectional area of 1750 mm^2. Portion AC is made of a mild steel with E = 200 GPa and σ_Y = 250 MPa, and portion BC is made of a high-strength steel with E = 200 GPa and σ_Y = 345 MPa. A load **P** is applied at C as shown. Assuming both steels to be elastoplastic, determine (a) the maximum deflection of C if P is gradually increased from zero to 975 kN and then reduced back to zero, (b) the maximum stress in each portion of the rod, (c) the permanent deflection of C.

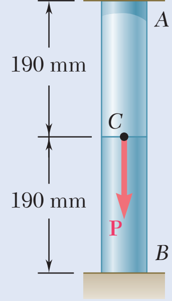


Fig. P2.107

2.108 For the composite rod of Prob. 2.107, if P is gradually increased from zero until the deflection of point C reaches a maximum value of δ_m = 0.3 mm and then decreased back to zero, determine, (a) the maximum value of P, (b) the maximum stress in each portion of the rod, (c) the permanent deflection of C after the load is removed.

2.109 Each cable has a cross-sectional area of 100 mm^2 and is made of an elastoplastic material for which σ_Y = 345 MPa and E = 200 GPa. A force **Q** is applied at C to the rigid bar ABC and is gradually increased from 0 to 50 kN and then reduced to zero. Knowing that the cables were initially taut, determine (a) the maximum stress that occurs in cable BD, (b) the maximum deflection of point C, (c) the final displacement of point C. (*Hint:* In part c, cable CE is not taut.)

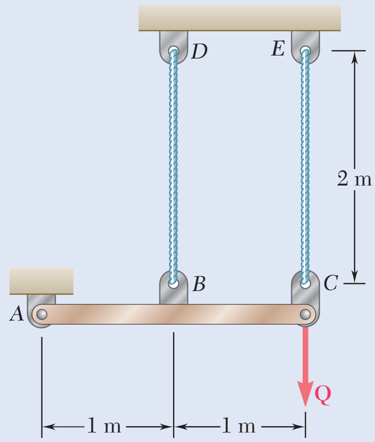


Fig. *P2.109*

2.110 Solve Prob. 2.109, assuming that the cables are replaced by rods of the same cross-sectional area and material. Further assume that the rods are braced so that they can carry compressive forces.

2.111 Two tempered-steel bars, each $\frac{3}{16}$ in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29 \times 10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value δ_m = 0.04 in. and then decreased back to zero. Determine (a) the maximum value of P, (b) the maximum stress in the tempered-steel bars, (c) the permanent set after the load is removed.

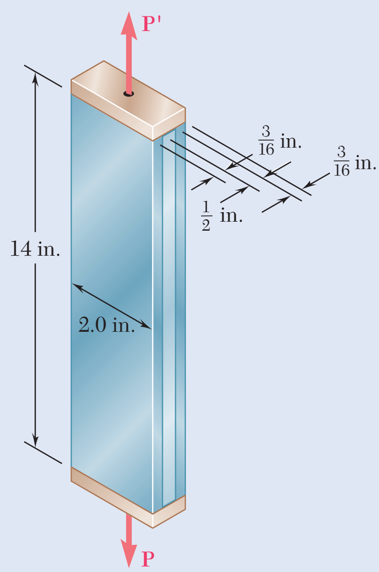


Fig. P2.111

from zero to 98 kips and then decreased back to zero, determine (*a*) the maximum deformation of the bar, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

2.113 The rigid bar *ABC* is supported by two links, *AD* and *BE*, of uniform 37.5 × 6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at *B* is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point *B*.

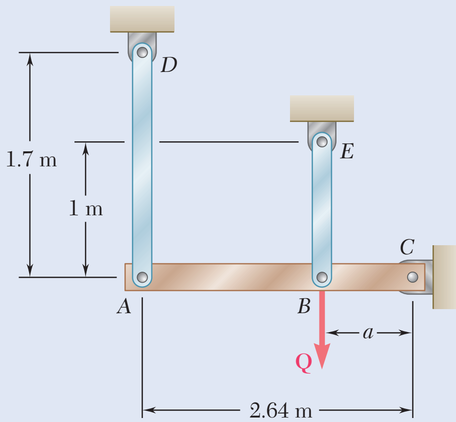


Fig. P2.113

2.114 Solve Prob. 2.113, knowing that $a = 1.76$ m and that the magnitude of the force **Q** applied at *B* is gradually increased from zero to 135 kN.

***2.115** Solve Prob. 2.113, assuming that the magnitude of the force **Q** applied at *B* is gradually increased from zero to 260 kN and then decreased back to zero. Knowing that $a = 0.640$ m, determine (*a*) the residual stress in each link, (*b*) the final deflection of point *B*. Assume that the links are braced so that they can carry compressive forces without buckling.

2.116 A uniform steel rod of cross-sectional area *A* is attached to rigid supports and is unstressed at a temperature of 45°F. The steel is assumed to be elastoplastic with $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi. Knowing that $\alpha = 6.5 \times 10^{-6}$/°F, determine the stress in the bar (*a*) when the temperature is raised to 320°F, (*b*) after the temperature has returned to 45°F.

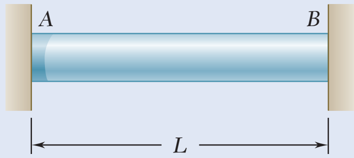


Fig. P2.116

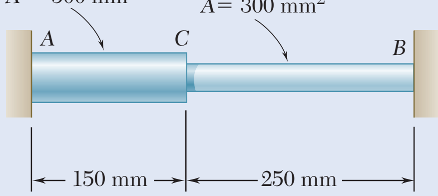


Fig. P2.117

at a temperature of 25°C. The steel is assumed elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The temperature of both portions of the rod is then raised to 150°C. Knowing that $\alpha = 11.7 \times 10^{-6}$/°C, determine (*a*) the stress in both portions of the rod, (*b*) the deflection of point *C*.

***2.118** Solve Prob. 2.117, assuming that the temperature of the rod is raised to 150°C and then returned to 25°C.

****2.119*** For the composite bar of Prob. 2.111, determine the residual stresses in the tempered-steel bars if *P* is gradually increased from zero to 98 kips and then decreased back to zero.

****2.120*** For the composite bar in Prob. 2.111, determine the residual stresses in the tempered-steel bars if *P* is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and is then decreased back to zero.

***2.121** Narrow bars of aluminum are bonded to the two sides of a thick steel plate as shown. Initially, at $T_1 = 70$°F, all stresses are zero. Knowing that the temperature will be slowly raised to T_2 and then reduced to T_1, determine (*a*) the highest temperature T_2 that does *not* result in residual stresses, (*b*) the temperature T_2 that will result in a residual stress in the aluminum equal to 58 ksi. Assume $\alpha_a = 12.8 \times 10^{-6}$/°F for the aluminum and $\alpha_s = 6.5 \times 10^{-6}$/°F for the steel. Further assume that the aluminum is elastoplastic with $E = 10.9 \times 10^6$ psi and $\alpha_Y = 58$ ksi. (*Hint:* Neglect the small stresses in the plate.)

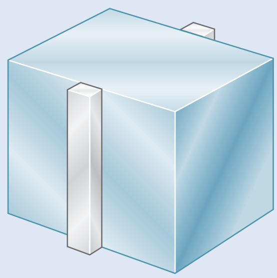


Fig. P2.121

***2.122** Bar *AB* has a cross-sectional area of 1200 mm² and is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. Knowing that the force **F** increases from 0 to 520 kN and then decreases to zero, determine (*a*) the permanent deflection of point *C*, (*b*) the residual stress in the bar.

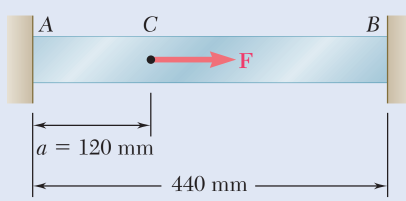


Fig. P2.122

***2.123** Solve Prob. 2.122, assuming that $a = 180$ mm.

Review and Summary

Normal Strain

Consider a rod of length L and uniform cross section, and its deformation δ under an axial load **P** (Fig. 2.59). The *normal strain* ϵ in the rod is defined as the *deformation per unit length*:

$$\epsilon = \frac{\delta}{L} \tag{2.1}$$

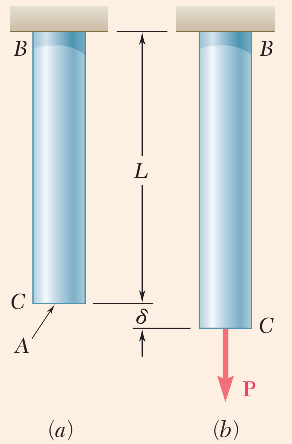


Fig. 2.59 Undeformed and deformed axially-loaded rod.

In the case of a rod of variable cross section, the normal strain at any given point Q is found by considering a small element of rod at Q:

$$\epsilon = \lim_{\Delta x \to 0} \frac{\Delta \delta}{\Delta x} = \frac{d\delta}{dx} \tag{2.2}$$

Stress-Strain Diagram

A *stress-strain diagram* is obtained by plotting the stress σ versus the strain ϵ as the load increases. These diagrams can be used to distinguish between *brittle* and *ductile* materials. A brittle material ruptures without any noticeable prior change in the rate of elongation (Fig. 2.60), while a ductile material

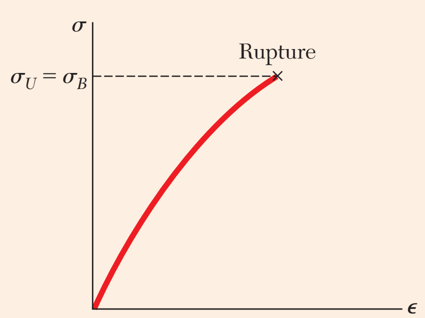
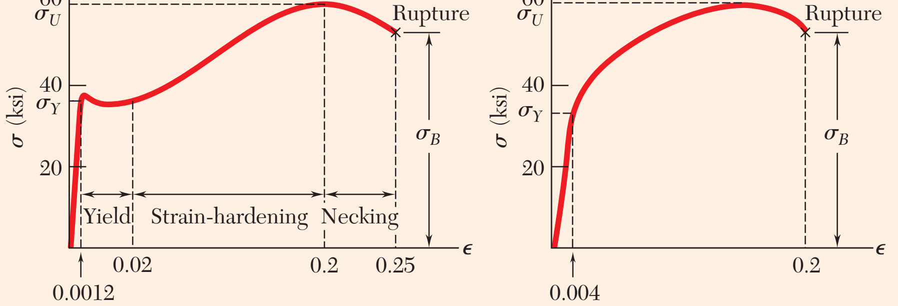


Fig. 2.60 Stress-strain diagram for a typical brittle material.



(*a*) Low-carbon steel (*b*) Aluminum alloy

Fig. 2.61 Stress-strain diagrams of two typical ductile metal materials.

yields after a critical stress σ_Y (the *yield strength*) has been reached (Fig. 2.61). The specimen undergoes a large deformation before rupturing, with a relatively small increase in the applied load. An example of brittle material with different properties in tension and compression is *concrete*.

Hooke's Law and Modulus of Elasticity

The initial portion of the stress-strain diagram is a straight line. Thus, for small deformations, the stress is directly proportional to the strain:

$$\sigma = E\epsilon \qquad \textbf{(2.6)}$$

This relationship is *Hooke's law*, and the coefficient E is the *modulus of elasticity* of the material. The *proportional limit* is the largest stress for which Eq. (2.4) applies.

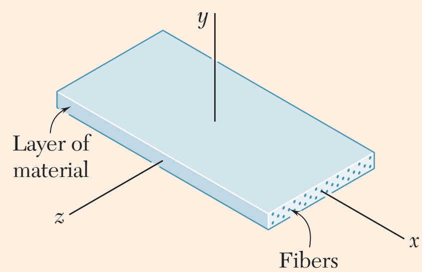


Fig. 2.62 Layer of fiber-reinforced composite material.

Properties of *isotropic* materials are independent of direction, while properties of *anisotropic* materials depend upon direction. *Fiber-reinforced composite materials* are made of fibers of a strong, stiff material embedded in layers of a weaker, softer material (Fig. 2.62).

Elastic Limit and Plastic Deformation

If the strains caused in a test specimen by the application of a given load disappear when the load is removed, the material is said to behave *elastically*. The largest stress for which this occurs is called the *elastic limit* of the material. If the elastic limit is exceeded, the stress and strain decrease in a linear fashion when the load is removed, and the strain does not return to zero (Fig. 2.63), indicating that a *permanent set* or *plastic deformation* of the material has taken place.

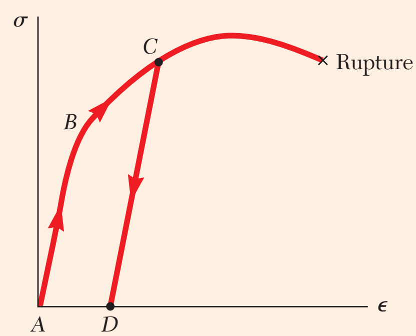


Fig. 2.63 Stress-strain response of ductile material loaded beyond yield and unloaded.

Fatigue causes the failure of structural or machine components after a very large number of repeated loadings, even though the stresses remain in the elastic range. A standard fatigue test determines the number n of successive loading-and-unloading cycles required to cause the failure of a specimen for any given maximum stress level σ and plots the resulting σ-n curve. The value of σ for which failure does not occur, even for an indefinitely large number of cycles, is known as the *endurance limit*.

Elastic Deformation Under Axial Loading

If a rod of length L and uniform cross section of area A is subjected at its end to a centric axial load **P** (Fig. 2.64), the corresponding deformation is

$$\delta = \frac{PL}{AE} \tag{2.9}$$

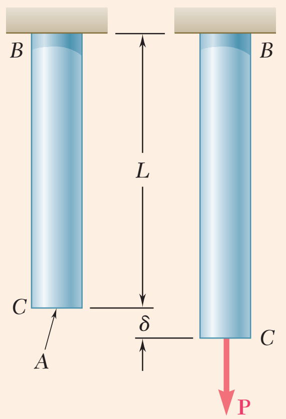


Fig. 2.64 Undeformed and deformed axially-loaded rod.

If the rod is loaded at several points or consists of several parts of various cross sections and possibly of different materials, the deformation δ of the rod must be expressed as the sum of the deformations of its component parts:

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i} \tag{2.10}$$

Statically Indeterminate Problems

Statically indeterminate problems are those in which the reactions and the internal forces *cannot* be determined from statics alone. The equilibrium equations derived from the free-body diagram of the member under consideration were complemented by relations involving deformations and obtained from the geometry of the problem. The forces in the rod and in the tube of Fig. 2.65, for instance, were determined by observing that their sum is equal to P, and that they cause equal deformations in the rod and in the tube. Similarly, the reactions at the supports of the bar of

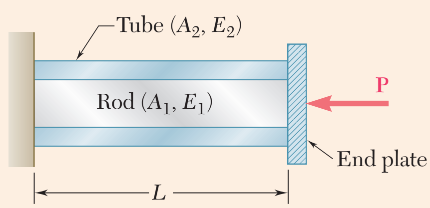


Fig. 2.65 Statically indeterminate problem w concentric rod and tube have same strain but different stresses.

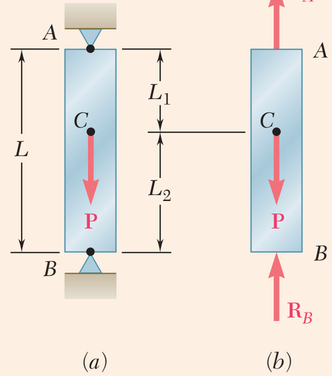


Fig. 2.66 (*a*) Axially-loaded statically-indeterminate member. (*b*) Free-body diagram.

Fig. 2.66 could not be obtained from the free-body diagram of the bar alone, but they could be determined by expressing that the total elongation of the bar must be equal to zero.

Problems with Temperature Changes

When the temperature of an *unrestrained rod AB* of length L is increased by ΔT, its elongation is

$$\delta_T = \alpha(\Delta T)L \tag{2.13}$$

where α is the *coefficient of thermal expansion* of the material. The corresponding strain, called *thermal strain*, is

$$\epsilon_T = \alpha \Delta T \tag{2.14}$$

and *no stress* is associated with this strain. However, if rod AB is *restrained* by fixed supports (Fig. 2.67), stresses develop in the rod as the temperature increases, because of the reactions at the supports. To determine the magnitude P of the reactions, the rod is first detached from its support at B (Fig. 2.68*a*).

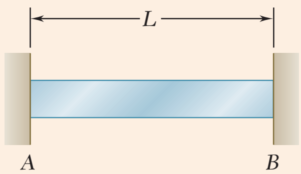


Fig. 2.67 Fully restrained bar of length *L*.

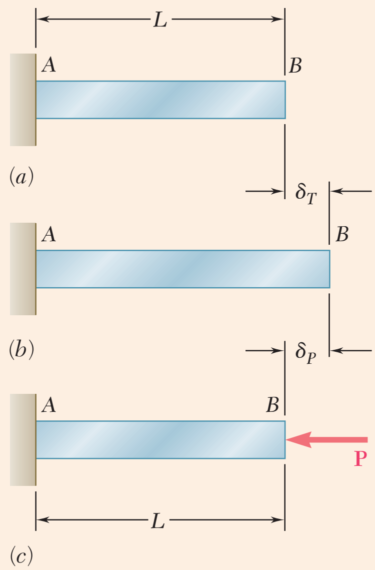


Fig. 2.68 Determination of reactions for bar of Fig. 2.67 subject to a temperature increase. (*a*) Support at *B* removed. (*b*) Thermal expansion. (*c*) Application of support reaction to counter thermal expansion.

change (Fig. 2.68*b*). The deformation δ_P caused by the force **P** is required to bring it back to its original length, so that it may be reattached to the support at *B* (Fig. 2.68*c*).

Lateral Strain and Poisson's Ratio

When an axial load **P** is applied to a homogeneous, slender bar (Fig. 2.69), it causes a strain, not only along the axis of the bar but in any transverse direction. This strain is the *lateral strain*, and the ratio of the lateral strain over the axial strain is called *Poisson's ratio*:

$$\nu = -\frac{\text{lateral strain}}{\text{axial strain}} \tag{2.17}$$

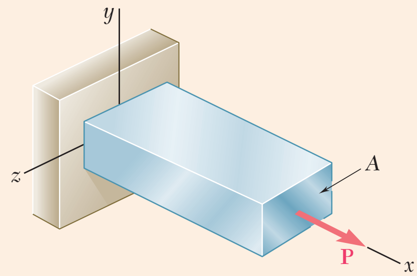


Fig. 2.69 A bar in uniaxial tension.

Multiaxial Loading

The condition of strain under an axial loading in the x direction is

$$\epsilon_x = \frac{\sigma_x}{E} \qquad \epsilon_y = \epsilon_z = -\frac{\nu\sigma_x}{E} \tag{2.19}$$

A *multiaxial loading* causes the state of stress shown in Fig. 2.70. The resulting strain condition was described by the *generalized Hooke's law* for a multiaxial loading.

$$\begin{aligned}
\epsilon_x &= +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E} \\
\epsilon_y &= -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E} \\
\epsilon_z &= -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}
\end{aligned} \tag{2.20}$$

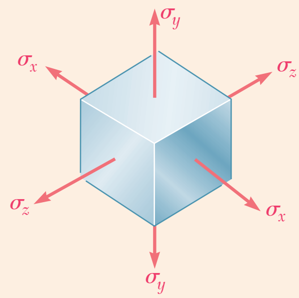


Fig. 2.70 State of stress for multiaxial loadin

Dilatation

If an element of material is subjected to the stresses σ_x, σ_y, σ_z, it will deform and a certain change of volume will result. The *change in volume per unit volume* is the *dilatation* of the material:

$$e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) \tag{2.22}$$

Bulk Modulus

When a material is subjected to a hydrostatic pressure p,

$$e = -\frac{p}{k} \tag{2.25}$$

where k is the *bulk modulus* of the material:

$$k = \frac{E}{3(1-2\nu)} \tag{2.24}$$

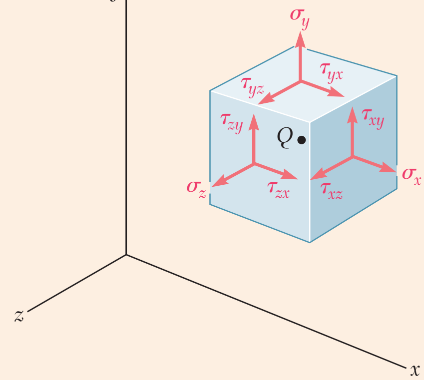


Fig. 2.71 Positive stress components at point Q for a general state of stress.

Shearing Strain: Modulus of Rigidity

The state of stress in a material under the most general loading condition involves shearing stresses, as well as normal stresses (Fig. 2.71). The shearing stresses tend to deform a cubic element of material into an oblique parallelepiped. The stresses τ_{xy} and τ_{yx} shown in Fig. 2.72 cause the angles formed by the faces on which they act to either increase or decrease by a small angle γ_{xy}. This angle defines the *shearing strain* corresponding to the x and y directions. Defining in a similar way the shearing strains γ_{yz} and γ_{zx}, the following relations were written:

$$\tau_{xy} = G\gamma_{xy} \qquad \tau_{yz} = G\gamma_{yz} \qquad \tau_{zx} = G\gamma_{zx} \qquad \textbf{(2.27, 28)}$$

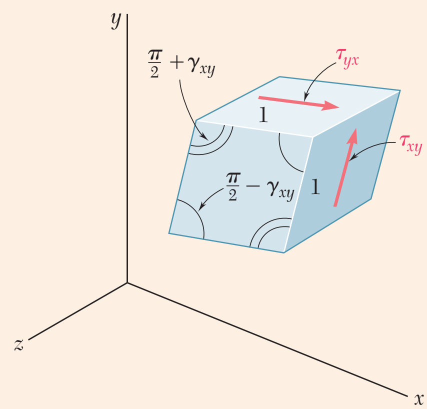


Fig. 2.72 Deformation of unit cubic element due to shearing stress.

which are valid for any homogeneous isotropic material within its proportional limit in shear. The constant G is the *modulus of rigidity* of the material, and the relationships obtained express *Hooke's law for shearing stress and strain*. Together with Eqs. (2.20), they form a group of equations representing the generalized Hooke's law for a homogeneous isotropic material under the most general stress condition.

While an axial load exerted on a slender bar produces only normal strains—both axial and transverse—on an element of material oriented

on an element rotated through 45° (Fig. 2.73). The three constants E, ν, and G are not independent. They satisfy the relation

$$\frac{E}{2G} = 1 + \nu \qquad \textbf{(2.34)}$$

This equation can be used to determine any of the three constants in terms of the other two.

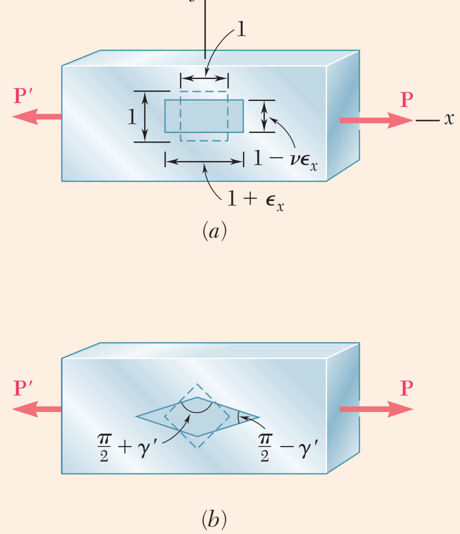


Fig. 2.73 Representations of strain in an axially-loaded bar: (*a*) cubic strain element wit faces aligned with coordinate axes; (*b*) cubic st element with faces rotated 45° about *z*-axis.

Saint-Venant's Principle

Saint-Venant's principle states that except in the immediate vicinity of the points of application of the loads, the distribution of stresses in a given member is independent of the actual mode of application of the loads. This principle makes it possible to assume a uniform distribution of stresses in a member subjected to concentrated axial loads, except close to the points of application of the loads, where stress concentrations will occur.

Stress Concentrations

Stress concentrations will also occur in structural members near a discontinuity, such as a hole or a sudden change in cross section. The ratio of the maximum value of the stress occurring near the discontinuity over the average stress computed in the critical section is referred to as the *stress-concentration factor* of the discontinuity:

$$K = \frac{\sigma_{\max}}{\sigma_{\text{ave}}} \qquad \textbf{(2.40)}$$

Plastic Deformations

Plastic deformations occur in structural members made of a ductile material when the stresses in some part of the member exceed the yield strength of the material. An idealized *elastoplastic material* is characterized by the stress-strain diagram shown in Fig. 2.74. When an indeterminate structure

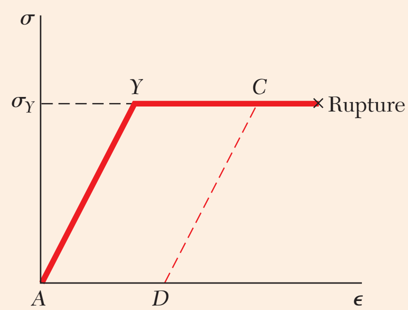


Fig. 2.74 Stress-strain diagram for an idealized elastoplastic material.

undergoes plastic deformations, the stresses do not, in general, return to zero after the load has been removed. The stresses remaining in the various parts of the structure are called *residual stresses* and can be determined by adding the maximum stresses reached during the loading phase and the reverse stresses corresponding to the unloading phase.

2.124 The uniform wire ABC, of unstretched length $2l$, is attached to the supports shown and a vertical load **P** is applied at the midpoint B. Denoting by A the cross-sectional area of the wire and by E the modulus of elasticity, show that, for $\delta << l$, the deflection at the midpoint B is

$$\delta = l\sqrt[3]{\frac{P}{AE}}$$

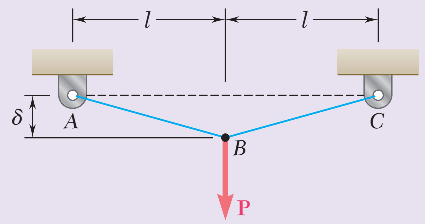


Fig. P2.124

2.125 The aluminum rod ABC ($E = 10.1 \times 10^6$ psi), which consists of two cylindrical portions AB and BC, is to be replaced with a cylindrical steel rod DE ($E = 29 \times 10^6$ psi) of the same overall length. Determine the minimum required diameter d of the steel rod if its vertical deformation is not to exceed the deformation of the aluminum rod under the same load and if the allowable stress in the steel rod is not to exceed 24 ksi.

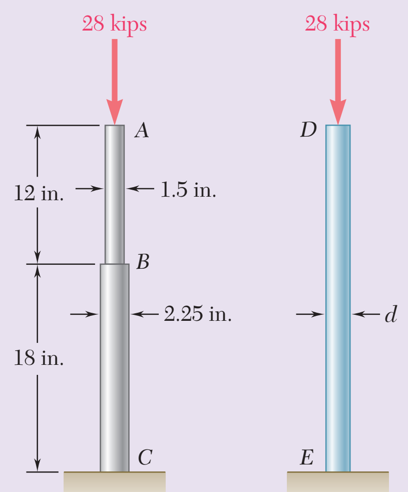


Fig. P2.125

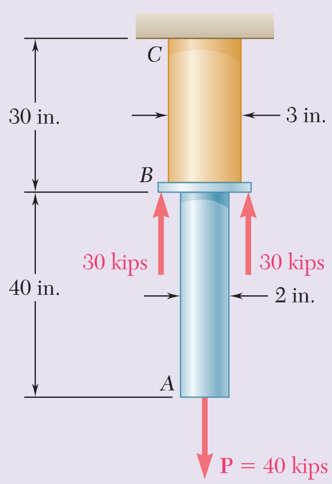


Fig. ***P2.126***

2.126 Two solid cylindrical rods are joined at B and loaded as shown. Rod AB is made of steel ($E = 29 \times 10^6$ psi), and rod BC of brass ($E = 15 \times 10^6$ psi). Determine (*a*) the total deformation of the composite rod ABC, (*b*) the deflection of point B.

and rests on a rough support at B. Knowing that the coefficient of friction is 0.60 between the strip and the support at B, determine the decrease in temperature for which slipping will impend.

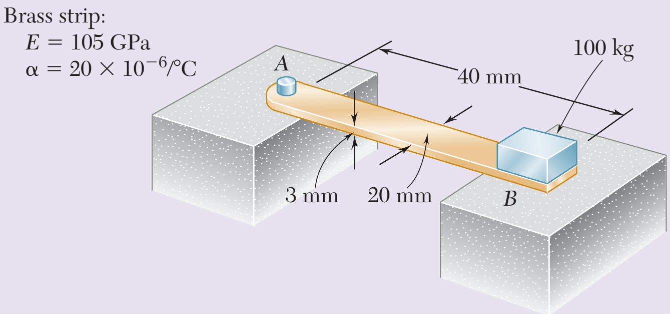


Fig. P2.127

2.128 The specimen shown is made from a 1-in.-diameter cylindrical steel rod with two 1.5-in.-outer-diameter sleeves bonded to the rod as shown. Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the load **P** so that the total deformation is 0.002 in., (*b*) the corresponding deformation of the central portion *BC*.

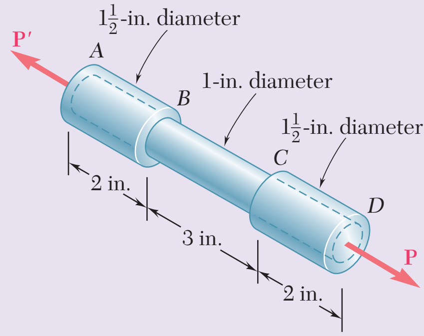


Fig. P2.128

2.129 Each of the four vertical links connecting the two rigid horizontal members is made of aluminum ($E = 70$ GPa) and has a uniform rectangular cross section of 10×40 mm. For the loading shown, determine the deflection of (*a*) point *E*, (*b*) point *F*, (*c*) point *G*.

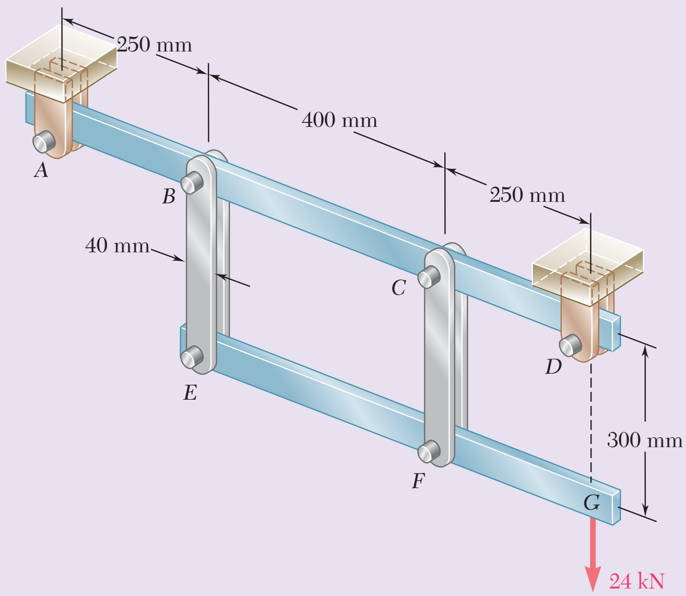


Fig. *P2.129*

2.130 A 4-ft concrete post is reinforced with four steel bars, each with a $\frac{3}{4}$-in. diameter. Knowing that $E_s = 29 \times 10^6$ psi and $E_c = 3.6 \times 10^6$ psi, determine the normal stresses in the steel and in the concrete when a 150-kip axial centric force **P** is applied to the post.

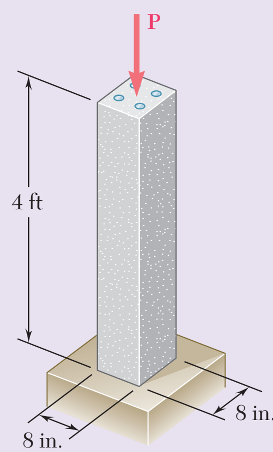


Fig. P2.130

(E = 200 GPa); the ends of the rods are single-threaded with a pitch of 2.5 mm. Knowing that after being snugly fitted, the nut at C is tightened one full turn, determine (a) the tension in rod CD, (b) the deflection of point C of the rigid member ABC.

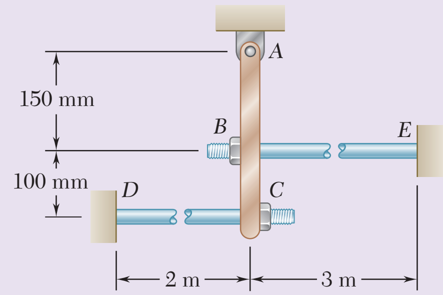


Fig. P2.131

2.132 The assembly shown consists of an aluminum shell (E_a = 10.6×10^6 psi, $\alpha_a = 12.9 \times 10^{-6}$/°F) fully bonded to a steel core ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}$/°F) and is unstressed. Determine (a) the largest allowable change in temperature if the stress in the aluminum shell is not to exceed 6 ksi, (b) the corresponding change in length of the assembly.

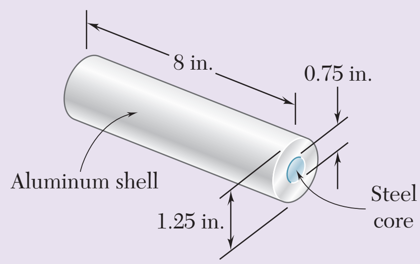


Fig. P2.132

2.133 The plastic block shown is bonded to a fixed base and to a horizontal rigid plate to which a force **P** is applied. Knowing that for the plastic used G = 55 ksi, determine the deflection of the plate when P = 9 kips.

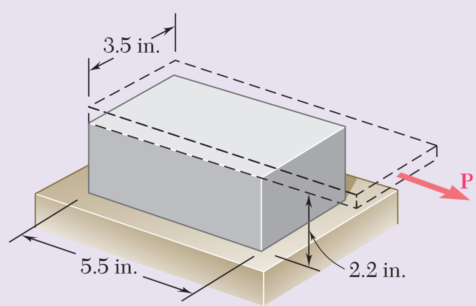


Fig. P2.133

and opposite centric axial forces of magnitude P. (*a*) Knowing that $E = 70$ GPa and $\sigma_{all} = 200$ MPa, determine the maximum allowable value of P and the corresponding total elongation of the specimen. (*b*) Solve part *a*, assuming that the specimen has been replaced by an aluminum bar of the same length and a uniform 60 × 15-mm rectangular cross section.

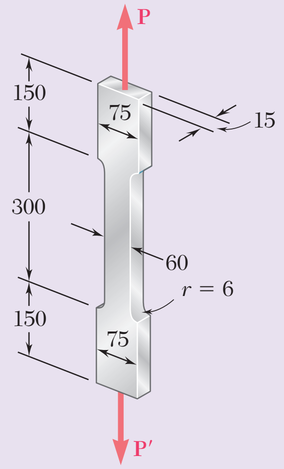


Fig. ***P2.134***

2.135 The uniform rod *BC* has cross-sectional area *A* and is made of a mild steel that can be assumed to be elastoplastic with a modulus of elasticity *E* and a yield strength σ_Y. Using the block-and-spring system shown, it is desired to simulate the deflection of end *C* of the rod as the axial force **P** is gradually applied and removed, that is, the deflection of points *C* and *C*′ should be the same for all values of *P*. Denoting by μ the coefficient of friction between the block and the horizontal surface, derive an expression for (*a*) the required mass *m* of the block, (*b*) the required constant *k* of the spring.

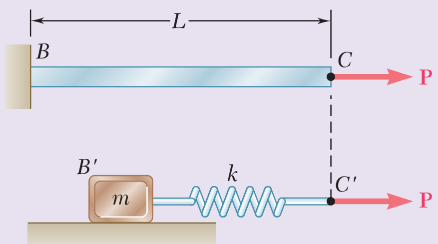


Fig. P2.135

Computer Problems

The following problems are designed to be solved with a computer. Write each program so that it can be used with either SI or U.S. customary units and in such a way that solid cylindrical elements may be defined by either their diameter or their cross-sectional area.

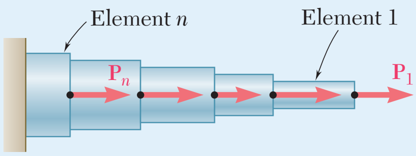


Fig. P2.C1

2.C1 A rod consisting of n elements, each of which is homogeneous and of uniform cross section, is subjected to the loading shown. The length of element i is denoted by L_i, its cross-sectional area by A_i, modulus of elasticity by E_i, and the load applied to its right end by $\mathbf{P}_i$, the magnitude P_i of this load being assumed to be positive if $\mathbf{P}_i$ is directed to the right and negative otherwise. (*a*) Write a computer program that can be used to determine the average normal stress in each element, the deformation of each element, and the total deformation of the rod. (*b*) Use this program to solve Probs. 2.20 and 2.126.

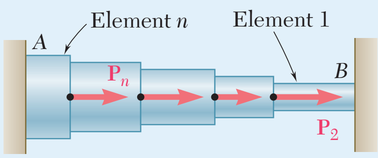


Fig. P2.C2

2.C2 Rod AB is horizontal with both ends fixed; it consists of n elements, each of which is homogeneous and of uniform cross section, and is subjected to the loading shown. The length of element i is denoted by L_i, its cross-sectional area by A_i, its modulus of elasticity by E_i, and the load applied to its right end by $\mathbf{P}_i$, the magnitude P_i of this load being assumed to be positive if $\mathbf{P}_i$ is directed to the right and negative otherwise. (Note that $P_1 = 0$.) (*a*) Write a computer program that can be used to determine the reactions at A and B, the average normal stress in each element, and the deformation of each element. (*b*) Use this program to solve Probs. 2.41 and 2.42.

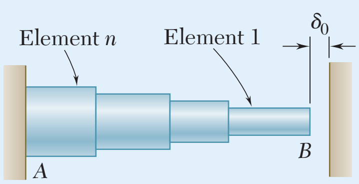


Fig. P2.C3

2.C3 Rod AB consists of n elements, each of which is homogeneous and of uniform cross section. End A is fixed, while initially there is a gap δ_0 between end B and the fixed vertical surface on the right. The length of element i is denoted by L_i, its cross-sectional area by A_i, its modulus of elasticity by E_i, and its coefficient of thermal expansion by α_i. After the temperature of the rod has been increased by ΔT, the gap at B is closed and the vertical surfaces exert equal and opposite forces on the rod. (*a*) Write a computer program that can be used to determine the magnitude of the reactions at A and B, the normal stress in each element, and the deformation of each element. (*b*) Use this program to solve Probs. 2.59 and 2.60.

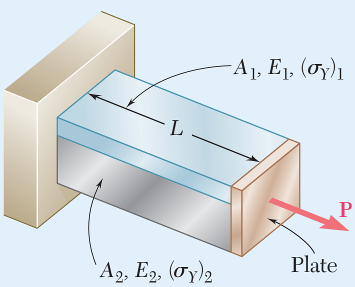


Fig. P2.C4

2.C4 Bar AB has a length L and is made of two different materials of given cross-sectional area, modulus of elasticity, and yield strength. The bar is subjected as shown to a load $\mathbf{P}$ that is gradually increased from zero until the deformation of the bar has reached a maximum value δ_m and then decreased back to zero. (*a*) Write a computer program that, for each of 25 values of δ_m equally spaced over a range extending from 0 to a value equal to 120% of the deformation causing both materials to yield, can be used to determine the maximum value P_m of the load, the maximum normal stress in each material, the permanent deformation δ_p of the bar, and the residual stress in each material. (*b*) Use this program to solve Probs. 2.111 and 2.112.